

# Performance analysis of an irreversible Brayton heat engine based on ecological coefficient of performance criterion

Yasin Ust<sup>a</sup>, Bahri Sahin<sup>a,\*</sup>, Ali Kodal<sup>b</sup>

<sup>a</sup> Department of Naval Architecture, Yildiz Technical University, Besiktas, 34349 Istanbul, Turkey

<sup>b</sup> Department of Aeronautical Engineering, Istanbul Technical University, Maslak, 34469 Istanbul, Turkey

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## Abstract

A performance optimization based on a new ecological criterion called ecological coefficient of performance (ECOP) has been presented for irreversible Brayton heat engine. The considered model includes irreversibilities due to finite rate heat transfer, heat leakage and internal dissipations. The ECOP objective function is defined as the ratio of power output to the loss rate of availability. The optimal performance and design parameters at maximum ECOP conditions are obtained analytically. The effects of major parameters on the general and optimal ecological performances have been investigated. The obtained results based on ECOP criterion are compared with an alternative ecological objective function defined in the literature and the maximum power output conditions, in terms of entropy generation rate, thermal efficiency and power output.

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**Keywords:** Brayton heat engine; Irreversibility; Ecological optimization; Performance analysis

## 1. Introduction

The classical Brayton cycle of the thermodynamics had been an extensive research focus due to its application importance to gas-turbine power plants in the industry. Some of the important studies related to the analysis presented here can start with Leff [1], who showed that the reversible heat engines based on Brayton, Otto, Diesel and Atkinson cycles operating at the maximum work output have efficiencies equal to Chambadal, Novikov and Curzon–Ahlborn efficiency [2–4]. Bejan [5] studied conductance allocation ratio of a Brayton cycle and showed the power is maximum for evenly distribution. Some other detailed works includes Wu and Kiang [6,7], Ibrahim et al. [8] and Feidt [9]. Sahin et al. [10–12] introduced maximum power density criterion for Brayton heat engines that leads smaller size and higher thermal efficiency. The optimization works in the literature

based on different objective functions for Brayton and the other heat engine models were reviewed by Bejan [13], Chen et al. [14] and recently by Durmayaz et al. [15].

Angulo-Brown [16] proposed an ecological optimization criterion by taking the objective function as  $E = \dot{W} - T_L \dot{S}_g$ , where  $\dot{W}$  is the power output,  $\dot{S}_g$  is the entropy generation rate and  $T_L$  is the temperature of the cold reservoir. Yan [17] discussed the result of Angulo-Brown [16] and suggested that it may be more reasonable to use  $E = \dot{W} - T_0 \dot{S}_g$  when the temperature of cold reservoir ( $T_L$ ) is not equal to the environment temperature ( $T_0$ ). Angulo-Brown [16] also showed that the efficiency at the maximum ecological function conditions is almost equal to the average of Carnot efficiency and the efficiency at maximum power conditions for an endoreversible Carnot heat engine. Cheng and Chen [18] performed an ecological optimization study based on the ecological criterion introduced by Angulo-Brown for endoreversible closed Brayton heat engine. They presented the ecologically optimum values of the power output and the corresponding thermal efficiency. Chen and Cheng [19] also investigated optimal design and performance parameters of

\* Corresponding author.

E-mail address: [sahinb@yildiz.edu.tr](mailto:sahinb@yildiz.edu.tr) (B. Sahin).

### Nomenclature

$A$	heat transfer area . . . . .	$m^2$	$\xi$	percentage of internal conductance
$\dot{C}_I$	internal conductance of the heat engine . . . . .	$kW \cdot K^{-1}$	<i>Subscripts</i>	
$\dot{C}_W$	product of mass flow rate and specific heat capacity, $= \dot{m}C_p$ . . . . .	$kW \cdot K^{-1}$	C	compressor
$\dot{E}$	ecological performance function		g	generation
ECOP	ecological coefficient of performance		H	high temperature heat source
$\dot{m}$	mass flow rate of the working fluid . . . .	$kg \cdot s^{-1}$	I	internal
$N_T$	total number of heat transfer units		L	low temperature heat source
$\dot{Q}$	rate of heat transfer . . . . .	$kW$	LK	heat leakage
$S$	entropy . . . . .	$kJ \cdot K^{-1}$	max	maximum
$T$	temperature . . . . .	$K$	mef	maximum ecological function conditions
$U$	overall heat transfer coefficient . . . . .	$kW \cdot m^{-2} \cdot K^{-1}$	mp	maximum power conditions
$\dot{W}$	power output . . . . .	$kW$	opt	optimum
<i>Greek symbols</i>				
$\chi$	allocation ratio, $= N_H / (N_L + N_H)$		T	turbine
$\varepsilon$	heat exchanger effectiveness		W	working fluid
$\phi$	isentropic temperature ratio		0	environment conditions
$\eta$	thermal efficiency		<i>Superscripts</i>	
$\tau$	heat sources temperature ratio, $= T_H / T_L$		*	maximum ECOP conditions
			–	dimensionless

an irreversible Brayton heat engine at the maximum ecological objective function condition. They concluded that the thermal conductance of the cold-side heat exchanger should be larger than that of the hot-side heat exchanger to achieve a higher ecological objective performance. They also obtained the optimum thermal conductance ratio and the isentropic temperature ratio parameter. Ust et al. [20] performed an ecological optimization for an endoreversible regenerative Brayton heat engine model by using the ecological optimization technique introduced by Angulo-Brown [16]. They investigated the effects of the regenerator effectiveness on the global and optimal performances and they also compared the performance at the maximum ecological function conditions with those of the maximum power conditions. They demonstrated the design conditions at the maximum ecological objective function may have more advantageous in terms of thermal efficiency, entropy generation rate and the investment cost although it may suffer the disadvantage of power loss in comparison to the design conditions at the maximum power output. The advantages at the maximum ecological function conditions increase as the regenerator effectiveness increases.

When we examine the results of the above optimization studies based on the ecological objective function proposed by Angulo-Brown [16] and improved by Yan [17], we see that the objective function may take negative values. This means that the loss rate of availability term,  $T_0 \dot{S}_g$ , is greater than the actual power output ( $\dot{W}$ ). The actual power output in the ecological objective function defined by Angulo-Brown [16] already includes the loss rate of availability. It should be noted that the actual power output equals to the

theoretical power output minus the loss rate of availability, i.e.,  $\dot{W} = \dot{W}_{th} - T_0 \dot{S}_g$  [21]. Therefore the loss rate of availability has been considered twice in the ecological objective function. In this context, this ecological objective function needs interpretation to comprehend the situation thermodynamically. Ust [22] has recently introduced a new dimensionless ecological optimization criterion called the ecological coefficient of performance (ECOP) that has always positive values and takes into account the loss rate of availability on the performance. The ECOP objective function is defined as the ratio of power output to the loss rate of availability, i.e.,  $ECOP = \dot{W} / T_0 \dot{S}_g$ . Using the ECOP criterion Ust et al. [23] performed a performance optimization study for an irreversible dual cycle.

In this paper, the new thermo-ecological optimization technique introduced by Ust [22] and Ust et al. [23] has been applied to an irreversible Brayton heat engine. To obtain the optimal performance and design parameters that maximize the ECOP objective function is major aim of this study. To fulfill the overall assessment of the performance of a Brayton heat engine in terms of ecology and also to compare the results with the other available techniques will be very beneficial for the engineering progress.

## 2. The theoretical model

The irreversible Brayton heat engine model and its  $T$ – $S$  diagram are shown in Fig. 1. The heat engine used in the model is operating between the heat source at high temperature,  $T_H$  and the heat sink at low temperature,  $T_L$ .  $\dot{Q}_H$  is

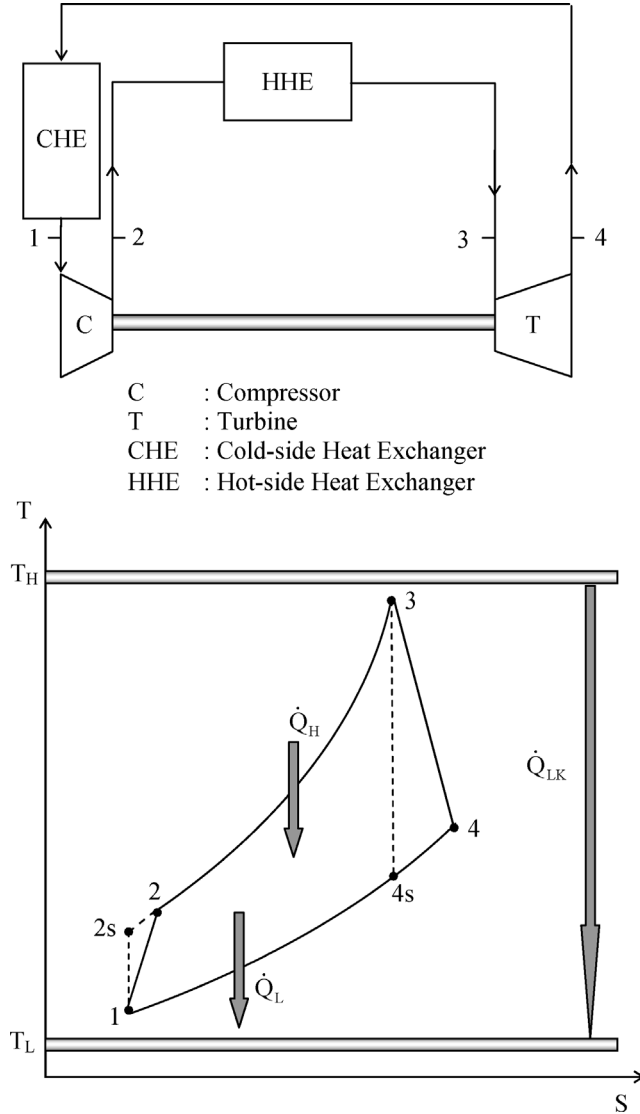


Fig. 1. Irreversible Brayton heat engine model and its  $T$ - $S$  diagram.

the rate of heat transferred from the heat source to the heat engine and  $\dot{Q}_L$  is the rate of heat dumped to the heat sink from the heat engine. Furthermore,  $U_H$  and  $U_L$  are overall heat transfer coefficients, and  $A_H$  and  $A_L$  are the heat transfer areas of the hot- and cold-side heat exchangers, respectively.  $\dot{C}_W$  is the thermal capacitance rate of the working fluid (ideal gas hypothesis with constant heat capacities is assumed). When the heat transfer obeys a linear law, equations of interest are

$$\dot{Q}_H = \dot{C}_W(T_3 - T_2) = \dot{C}_W \varepsilon_H(T_H - T_2) \quad (1)$$

$$\dot{Q}_L = \dot{C}_W(T_4 - T_1) = \dot{C}_W \varepsilon_L(T_4 - T_L) \quad (2)$$

where the effectiveness's of hot and cold-side heat exchangers  $\varepsilon_H$  and  $\varepsilon_L$  for counter-flow heat exchangers are defined as [24]

$$\varepsilon_H = 1 - e^{-N_H} \quad (3)$$

$$\varepsilon_L = 1 - e^{-N_L} \quad (4)$$

and the numbers of heat transfer units of hot- and cold-side heat exchangers are

$$N_H = (U_H A_H) / \dot{C}_W \quad (5)$$

$$N_L = (U_L A_L) / \dot{C}_W \quad (6)$$

Using Bejan's linear-model [5], the rate of heat leakage  $\dot{Q}_{LK}$  from the hot reservoir at temperature  $T_H$  to the cold reservoir at temperature  $T_L$  is given by

$$\dot{Q}_{LK} = \dot{C}_I(T_H - T_L) = \xi \dot{C}_W(T_H - T_L) \quad (7)$$

where  $\dot{C}_I$  is the internal conductance of the heat engine and  $\xi$  denotes the percentage of the internal conductance with respect to the thermal capacitance rate of the working fluid ( $\xi = \dot{C}_I / \dot{C}_W$ ). Then, the total heat rate  $\dot{Q}_{HT}$  transferred from the hot reservoir becomes

$$\begin{aligned} \dot{Q}_{HT} &= \dot{Q}_H + \dot{Q}_{LK} \\ &= \dot{C}_W \varepsilon_H(T_H - T_2) + \xi \dot{C}_W(T_H - T_L) \end{aligned} \quad (8)$$

and the total heat rate  $\dot{Q}_{LT}$  transferred to the cold reservoir is

$$\begin{aligned} \dot{Q}_{LT} &= \dot{Q}_L + \dot{Q}_{LK} \\ &= \dot{C}_W \varepsilon_L(T_4 - T_L) + \xi \dot{C}_W(T_H - T_L) \end{aligned} \quad (9)$$

The power produced by the Brayton heat engine according to the first law of thermodynamics can be given as

$$\begin{aligned} \dot{W} &= \dot{Q}_{HT} - \dot{Q}_{LT} = \dot{Q}_H - \dot{Q}_L \\ &= \dot{C}_W[\varepsilon_H(T_H - T_2) - \varepsilon_L(T_4 - T_L)] \end{aligned} \quad (10)$$

The thermal efficiency becomes

$$\eta = \frac{\dot{W}}{\dot{Q}_{HT}} = \frac{\varepsilon_H(T_H - T_2) - \varepsilon_L(T_4 - T_L)}{\varepsilon_H(T_H - T_2) + \xi(T_H - T_L)} \quad (11)$$

The entropy generation rate is

$$\begin{aligned} \dot{S}_g &= \frac{\dot{Q}_{LT}}{T_L} - \frac{\dot{Q}_{HT}}{T_H} \\ &= \dot{C}_W \left[ \frac{\varepsilon_L(T_4 - T_L)}{T_L} - \frac{\varepsilon_H(T_H - T_2)}{T_H} + \xi \frac{(\tau - 1)^2}{\tau} \right] \end{aligned} \quad (12)$$

where  $\tau$  is the ratio of high and the low source temperatures,  $\tau = T_H / T_L$ . Eqs. (1) and (2) yield

$$T_3 = \varepsilon_H T_H + T_2(1 - \varepsilon_H) \quad (13)$$

$$T_4 = \frac{T_1 - \varepsilon_L T_L}{(1 - \varepsilon_L)} \quad (14)$$

The isentropic efficiencies of turbine and compressor are

$$\eta_C = \frac{T_{2S} - T_1}{T_2 - T_1} \quad (15)$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4S}} \quad (16)$$

Eqs. (15) and (16) give

$$T_{2S} = (1 - \eta_C)T_1 + \eta_C T_2 \quad (17)$$

$$T_{4S} = T_4 / \eta_T + (1 - 1/\eta_T)T_3 \quad (18)$$

The second law of thermodynamics requires using perfect gas hypothesis that

$$\phi = T_{2s}/T_1 = T_3/T_{4s} \tag{19}$$

where  $\phi$  is the isentropic temperature ratio of the Brayton cycle. By using Eqs. (17)–(19), we get

$$T_2 = T_1 \left( \frac{\phi - 1 + \eta_C}{\eta_C} \right) \tag{20}$$

$$T_3 = T_1(1 - \varepsilon_H) \left( \frac{\phi - 1 + \eta_C}{\eta_C} \right) + \varepsilon_H T_H \tag{21}$$

By substituting Eqs. (13), (14), (17), (18), (20) and (21) into Eq. (19), we have

$$T_1 = \frac{a_1\phi + a_2}{a_3\phi^2 + a_4\phi + a_5} \tag{22}$$

Where the simplification parameters are defined as

$$a_1 = \varepsilon_H T_H \left( 1 - \frac{1}{\eta_T} \right) - \frac{\varepsilon_L T_L}{\eta_T(1 - \varepsilon_L)} \tag{23}$$

$$a_2 = -\varepsilon_H T_H \tag{24}$$

$$a_3 = \frac{1}{\eta_C} \left( 1 - \frac{1}{\eta_T} \right) (\varepsilon_H - 1) \tag{25}$$

$$a_4 = (1 - \varepsilon_H) \frac{1}{\eta_C} + \frac{1}{\eta_T(\varepsilon_L - 1)} + (\varepsilon_H - 1) \left( 1 - \frac{1}{\eta_T} \right) \left( 1 - \frac{1}{\eta_C} \right) \tag{26}$$

$$a_5 = (1 - \varepsilon_H) \left( 1 - \frac{1}{\eta_C} \right) \tag{27}$$

The ECOP (ecological coefficient of performance) objective function defined as the ratio of power output to the loss rate of availability can be written as

$$ECOP = \frac{\dot{W}}{T_0 \dot{S}_g} \tag{28}$$

where  $T_0$  is the environment temperature. Using Eqs. (10) and (12) in Eq. (28), the ECOP function is derived as

$$ECOP = \frac{\varepsilon_H(T_H - T_2) - \varepsilon_L(T_4 - T_L)}{T_0 \left[ \frac{\varepsilon_L(T_4 - T_L)}{T_L} - \frac{\varepsilon_H(T_H - T_2)}{T_H} + \xi \frac{(\tau - 1)^2}{\tau} \right]} \tag{29}$$

Substitutions of Eq. (22) into Eqs. (14) and (20) give

$$T_2 = \frac{(a_1\phi + a_2)}{(a_3\phi^2 + a_4\phi + a_5)} \left( \frac{\phi - 1 + \eta_C}{\eta_C} \right) \tag{30}$$

$$T_4 = \frac{(a_1\phi + a_2)}{(1 - \varepsilon_L)(a_3\phi^2 + a_4\phi + a_5)} - \frac{\varepsilon_L T_L}{(1 - \varepsilon_L)} \tag{31}$$

Finally, using Eqs. (30) and (31) in Eq. (29), the ECOP function can be regarded as simple algebraic relation, i.e.

$$ECOP = \frac{b_1\phi^2 + b_2\phi + b_3}{c_1\phi^2 + c_2\phi + c_3} \tag{32}$$

where

$$b_1 = a_1 a_6 + a_3 a_7 \tag{33}$$

$$b_2 = a_1 a_8 + a_2 a_6 + a_4 a_7 \tag{34}$$

$$b_3 = a_2 a_8 + a_5 a_7 \tag{35}$$

$$c_1 = a_1 a_9 + a_3 a_{10} \tag{36}$$

$$c_2 = a_1 a_{11} + a_2 a_9 + a_4 a_{10} \tag{37}$$

$$c_3 = a_2 a_{11} + a_5 a_{10} \tag{38}$$

and where the defined parameters  $a_6, a_7, a_8, a_9, a_{10}$  and  $a_{11}$  for simplicity:

$$a_6 = -\frac{\varepsilon_H}{\eta_C} \tag{39}$$

$$a_7 = \varepsilon_H T_H + \frac{\varepsilon_L T_L}{1 - \varepsilon_L} \tag{40}$$

$$a_8 = -\frac{\varepsilon_L}{1 - \varepsilon_L} - \varepsilon_H \left( 1 - \frac{1}{\eta_C} \right) \tag{41}$$

$$a_9 = \frac{T_0 \varepsilon_H}{T_H \eta_C} \tag{42}$$

$$a_{10} = T_0 \left[ \frac{\xi(\tau - 1)^2}{\tau} - \frac{\varepsilon_L^2}{1 - \varepsilon_L} - \varepsilon_L - \varepsilon_H \right] \tag{43}$$

$$a_{11} = T_0 \left[ \frac{\varepsilon_L}{T_L(1 - \varepsilon_L)} + \frac{\varepsilon_H}{T_H} \left( 1 - \frac{1}{\eta_C} \right) \right] \tag{44}$$

The variation of ECOP function with respect to the isentropic temperature ratio ( $\phi$ ), for different values of isentropic efficiencies ( $\eta_C, \eta_T$ ), heat sources temperature ratio ( $\tau$ ) and the total number of heat transfer units ( $N_T = N_H + N_L$ ) are demonstrated in Fig. 2(a)–(c). We can observe from the figure that the ECOP function has a maximum for a certain  $\phi$  value for chosen set of operation parameters. The maximum of ECOP with respect to  $\phi$  can be found analytically by setting  $dECOP/d\phi = 0$ . The optimum value of  $\phi$  at the maximum ECOP is found as:

$$\phi_{opt} = \frac{c_1 b_3 - b_1 c_3}{b_1 c_2 - c_1 b_2} \times \left[ 1 - \sqrt{1 - \frac{(b_2 c_3 - c_2 b_3)(b_1 c_2 - c_1 b_2)}{(c_1 b_3 - b_1 c_3)^2}} \right] \tag{45}$$

The maximum of ECOP can be obtained by substituting Eq. (45) into Eq. (32), i.e.

$$ECOP_{max} = \frac{b_1 \phi_{opt}^2 + b_2 \phi_{opt} + b_3}{c_1 \phi_{opt}^2 + c_2 \phi_{opt} + c_3} \tag{46}$$

Also, we can obtain the optimum value of power at the maximum ECOP, i.e.

$$\frac{\dot{W}_{opt}}{\dot{C}_W} = \frac{b_1 \phi_{opt}^2 + b_2 \phi_{opt} + b_3}{a_3 \phi_{opt}^2 + a_4 \phi_{opt} + a_5} \tag{47}$$

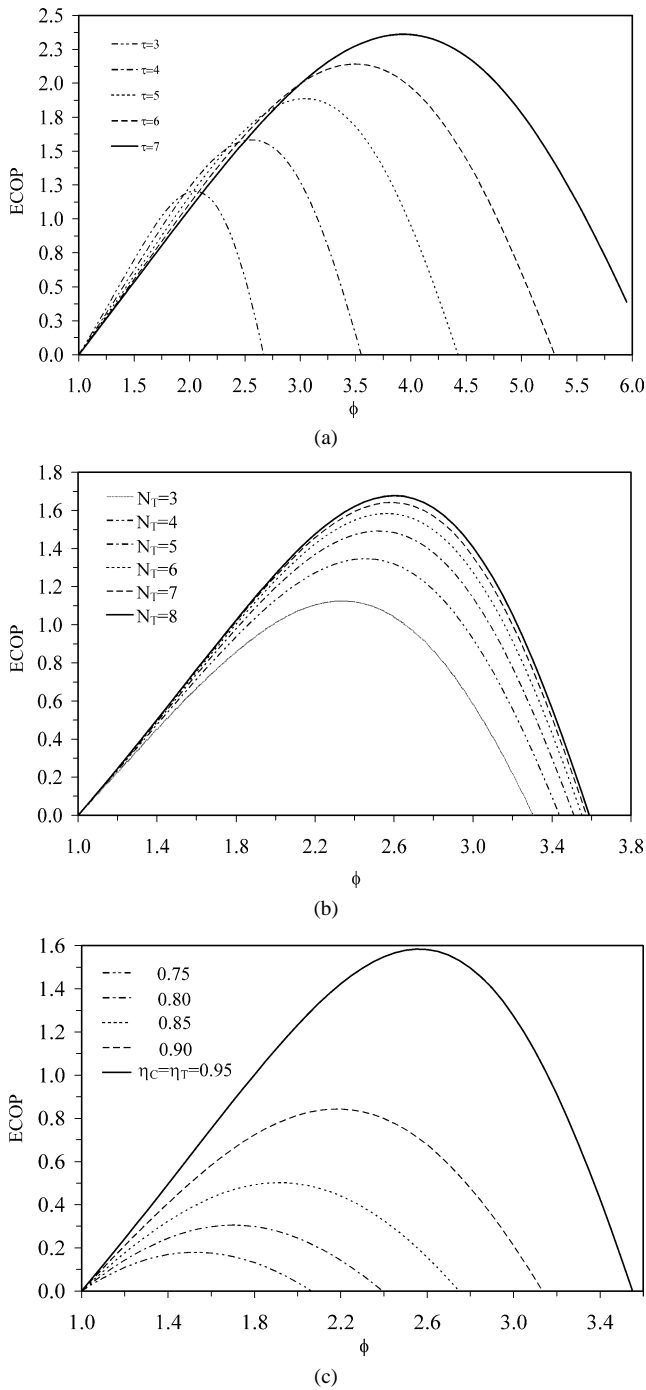


Fig. 2. Variation of the ECOP function with respect to the isentropic temperature ratio for various (a)  $\tau$ ; (b)  $N_T$  and (c)  $\eta_C = \eta_T$  values ( $T_L = 300$  K in all and the required constants are selected as  $T_H = 1200$  K,  $\xi = 0.02$ ,  $\eta_C = 0.9$ ,  $\eta_T = 0.95$ ,  $\varepsilon_L = \varepsilon_H = 0.9$ ).

### 3. Results and discussion

The maximum of the ECOP function signifies the importance of getting a power from a heat engine by causing lesser dissipation in the environment. Therefore the higher the ECOP, we have a better heat engine in terms of power and the environment considered together. In Fig. 3(a)–(c),

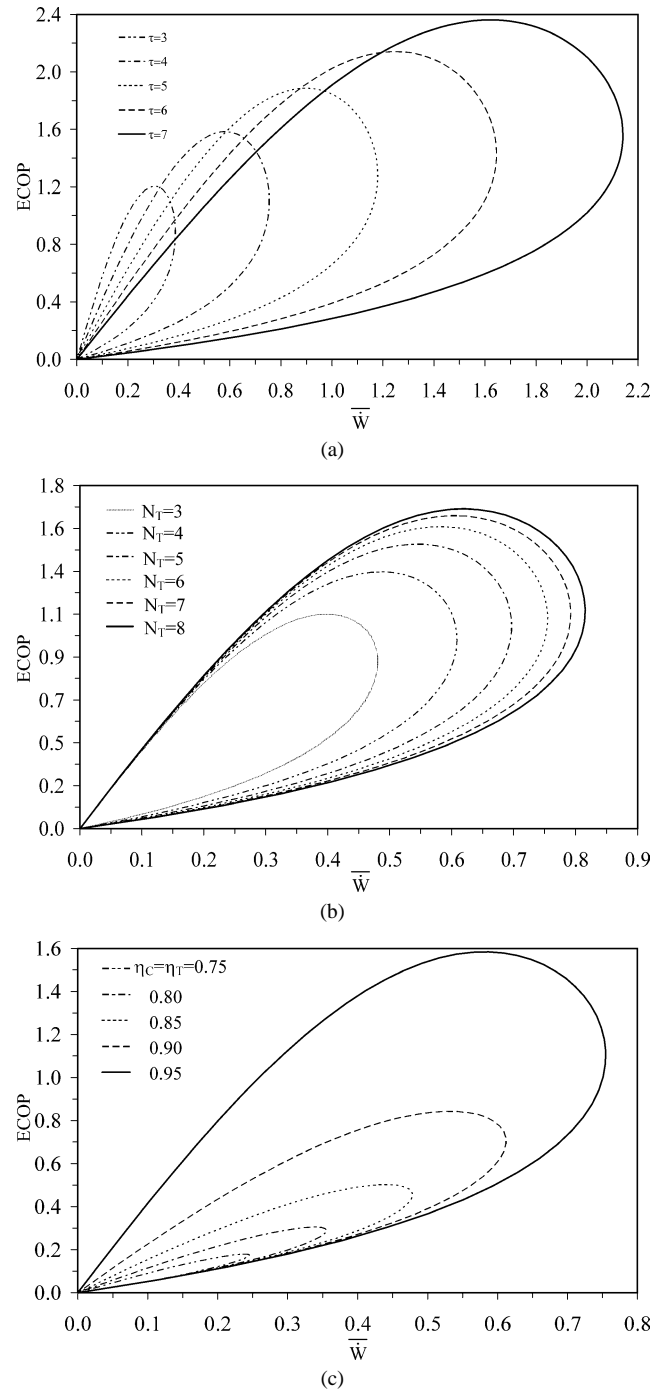


Fig. 3. Variation of the ECOP function with respect to the power output for various (a)  $\tau$ ; (b)  $N_T$  and (c)  $\eta_C = \eta_T$  values ( $T_L = 300$  K in all and the required constants are selected as  $T_H = 1200$  K,  $\xi = 0.02$ ,  $\eta_C = 0.9$ ,  $\eta_T = 0.95$ ,  $\varepsilon_L = \varepsilon_H = 0.9$ ).

we plotted the variations of ECOP function with respect to the dimensionless power output,  $[\bar{W} = \dot{W}/(\dot{C}_W T_L)]$  for different selected values of the heat sources temperature ratio ( $\tau$ ), total number of heat transfer units ( $N_T = N_L + N_H$ ) and isentropic efficiencies ( $\eta_C, \eta_T$ ). From these figures we can evaluate the effects of the design parameters on the ECOP and power for better ecology. The curves in Fig. 3(a)–(c) are

all in loop forms and thus they have two different maxima for both axes, which are ECOP and power respectively. The performances in terms of ECOP and  $\dot{W}$  decrease as  $\tau$ ,  $N_T$  and  $\eta_C = \eta_T$  decrease. Of course, the power output at the maximum ECOP conditions ( $\dot{W}^*$ ) is lower than the maximum power output ( $\dot{W}_{\max}$ ). But also, the ECOP at  $\dot{W}_{\max}$  conditions ( $\text{ECOP}_{\text{mp}}$ ) is lower than the  $\text{ECOP}_{\max}$ . So we need to evaluate a compromise between the power and the ecology. Also, we can observe from Fig. 3 that  $\dot{W}^*$  and  $\dot{W}_{\max}$ , or  $\text{ECOP}_{\max}$  and  $\text{ECOP}_{\text{mp}}$  get closer to each other for decreasing  $\tau$ ,  $N_T$  and  $\eta_C = \eta_T$ . It should be noted that, for a certain power generation requirements, the loss rate of availability,  $T_0 \dot{S}_g$  is minimum at  $\text{ECOP}_{\max}$  conditions. Therefore the maximization of the ECOP function represents the best compromise between the power output and the loss rate of availability for environmental aspects. By considering the power and entropy generation rate (or loss rate of availability) of the heat engine together, the optimal design intervals in terms of ECOP and  $\dot{W}$  will be

$$\text{ECOP}_{\max} \geq \text{ECOP} \geq \text{ECOP}_{\text{mp}}, \quad \text{or} \quad \dot{W}^* \leq \dot{W} \leq \dot{W}_{\max} \quad (48)$$

The choice of the optimal design parameters of a heat engine depends on working place and its purpose. That is, if the power is the major concern, the design parameters should be chosen close to  $\dot{W}_{\max}$  conditions, however, they may also be chosen close to  $\text{ECOP}_{\max}$  conditions for the minimum entropy generation rate (or minimum environmental damage caused by thermal pollution). For the engineering point of view, the maximum of thermal efficiency is also important. We can relate the ECOP and the thermal efficiency by using the above equations, i.e.

$$\text{ECOP} = \frac{\eta}{\frac{T_0}{T_L} (1 - \eta - \frac{1}{\tau})} \quad (49)$$

Therefore, the thermal efficiency and the ECOP function depend on each other for given extreme and environment temperatures. Furthermore, their maximums coincide although they have different definitions and so meanings. Getting the same performance conditions at the maximums of the ECOP and the thermal efficiency is an expected and logical result. Since the maximum thermal efficiency conditions for a certain power yields minimum fuel consumption so that minimum environment pollution, the coincidence with the maximum ECOP conditions mean that the defined ecological objective function (ECOP) considers the environment effect appropriately. The thermal efficiency gives information about the necessary fuel consumption in order to produce certain power while ECOP gives information about the entropy generation, i.e. the loss rate of availability. In Fig. 4(a), and (b) the comparisons of several objective functions, namely ECOP, dimensionless ecological function [ $\dot{E} = \dot{E}/(\dot{C}_W T_L)$ ] proposed by Angulo-Brown [16] and dimensionless power output ( $\bar{W}$ ) with respect to the isentropic temperature ratio,  $\phi$  and dimensionless entropy

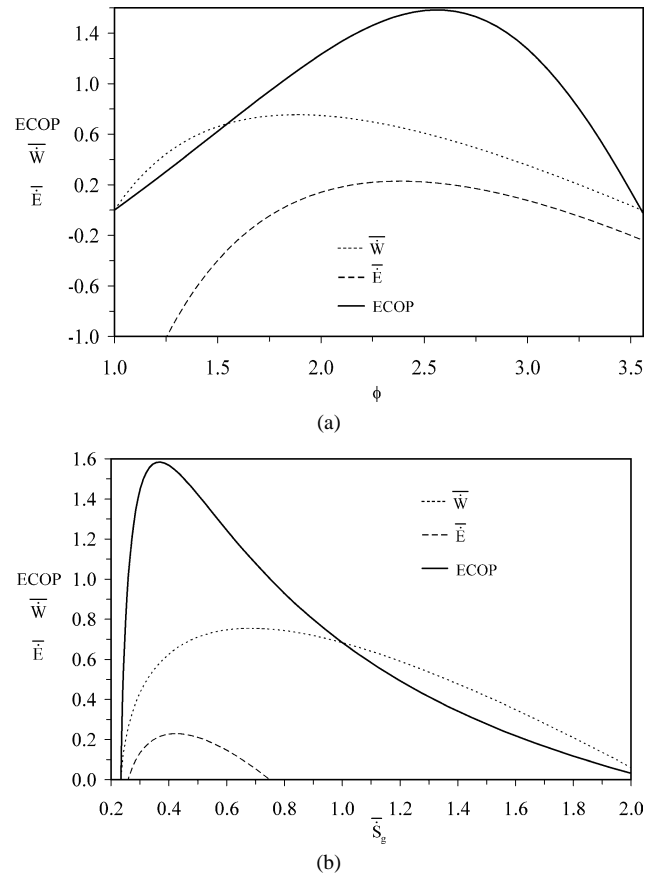


Fig. 4. Variations of the ECOP, dimensionless power output and the ecologic performance functions with respect to the (a) isentropic temperature ratio and (b) dimensionless entropy generation rate ( $T_L = 300$  K,  $T_H = 1200$  K,  $\xi = 0.02$ ,  $\eta_C = \eta_T = 0.95$ ,  $\varepsilon_L = \varepsilon_H = 0.95$ ).

generation rate, [ $\bar{S}_g = \dot{S}_g/\dot{C}_W$ ] are demonstrated. It is seen from Fig. 4(a) that the optimum isentropic temperature ratio at the  $\text{ECOP}_{\max}$  conditions ( $\phi^*$ ) is always greater than those of at  $\dot{E}_{\max}$  and  $\dot{W}_{\max}$  conditions ( $\phi_{\text{mef}}$ ,  $\phi_{\text{mp}}$ ), so we can write  $\phi^* > \phi_{\text{mef}} > \phi_{\text{mp}}$ . We observe from Fig. 4(b) that the entropy generation rate at the  $\text{ECOP}_{\max}$  conditions ( $\dot{S}_g^*$ ) is lower than the entropy generation rate at  $\dot{E}_{\max}$  conditions ( $\dot{S}_{g,\text{mef}}$ ) and at  $\dot{W}_{\max}$  conditions ( $\dot{S}_{g,\text{mp}}$ ), i.e.  $\dot{S}_g^* < \dot{S}_{g,\text{mef}} < \dot{S}_{g,\text{mp}}$ .

The effect of NTU allocation ratio ( $\chi = N_H/N_T$ ) on the  $\text{ECOP}_{\max}$  is also investigated and the results are presented in Fig. 5(a) and (b) for various  $N_T$  and  $\tau$  values. It is seen from these figures that the optimum value of NTU allocation ratio ( $\chi^*$ ) which yields the maximum of  $\text{ECOP}_{\max}$  is slightly affected by  $N_T$  and  $\tau$ . The effects of  $N_T$  and  $\tau$  on the both of  $\phi^*$  and  $\chi^*$  which are the important design parameters for a gas turbine, are shown in Fig. 6. As can be seen from this figure,  $\phi^*$  and  $\chi^*$  values increase as  $N_T$  increases for a specified  $\tau$  value. However for a specified  $N_T$  value, as  $\tau$  increases  $\chi^*$  decreases while  $\phi^*$  increases. When we examine Fig. 6, we see that the effect of  $\tau$  on the  $\phi^*$  is greater than on the  $\chi^*$  (for example, for  $N_T = 5$  as  $\tau$  varies between 3 and 7  $\phi^*$  changed 87 percent on the other hand  $\chi^*$  changed ap-

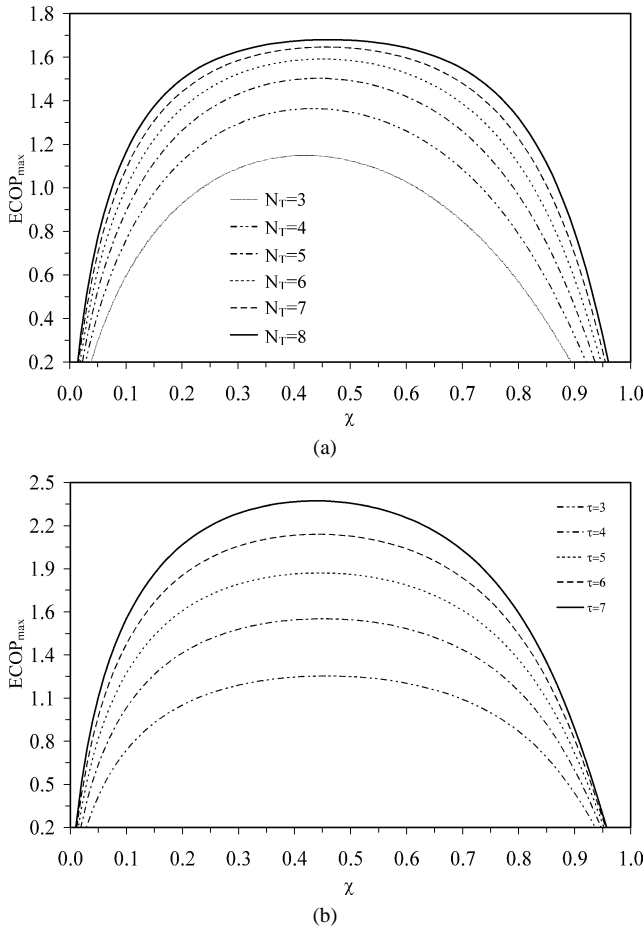


Fig. 5. Variation of the maximum ECOP with respect to NTU allocation ratio ( $\chi$ ) for various (a)  $N_T$  and (b)  $\tau$  values ( $T_L = 300$  K in all and the required constants are selected as  $T_H = 1200$  K,  $\xi = 0.02$ ,  $\eta_C = 0.9$ ,  $\eta_T = 0.95$ ,  $\varepsilon_L = \varepsilon_H = 0.9$ ).

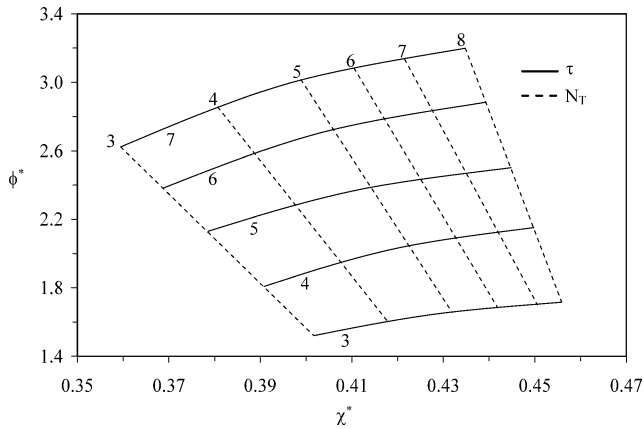


Fig. 6. Variation of the optimal isentropic temperature ratio ( $\phi^*$ ) with respect to optimal NTU allocation ratio ( $\chi^*$ ) for various  $\tau$  and  $N_T$  values ( $T_L = 300$  K,  $T_H = 1200$  K,  $\xi = 0.02$ ,  $\eta_C = 0.9$ ,  $\eta_T = 0.95$ ,  $\varepsilon_L = \varepsilon_H = 0.9$ ).

proximately 10 percent). The optimal performances in terms of  $\dot{W}$ ,  $\dot{S}_g$  and  $\eta$  for the  $ECOP_{max}$ ,  $E_{max}$  and  $\dot{W}_{max}$  conditions are compared in Figs. 7–9. It is clearly seen from these figures that a design conditions based on the maximum of

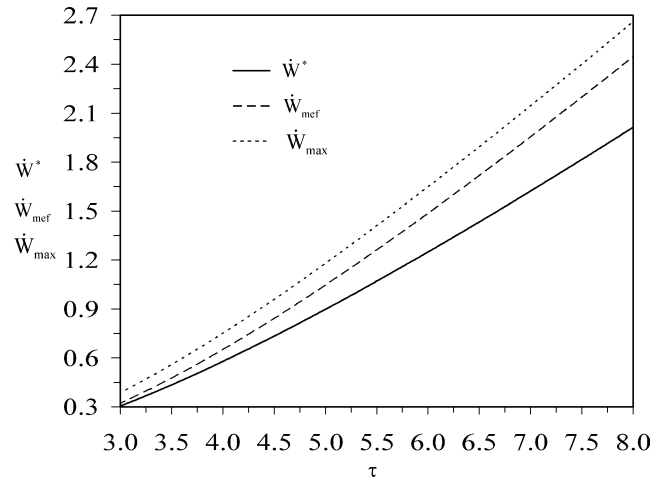


Fig. 7. The effect of source temperature ratio  $\tau$ , on the optimal power output values ( $T_L = 300$  K,  $T_H = 1200$  K,  $\xi = 0.02$ ,  $\eta_C = 0.9$ ,  $\eta_T = 0.95$ ,  $\varepsilon_L = \varepsilon_H = 0.9$ ).

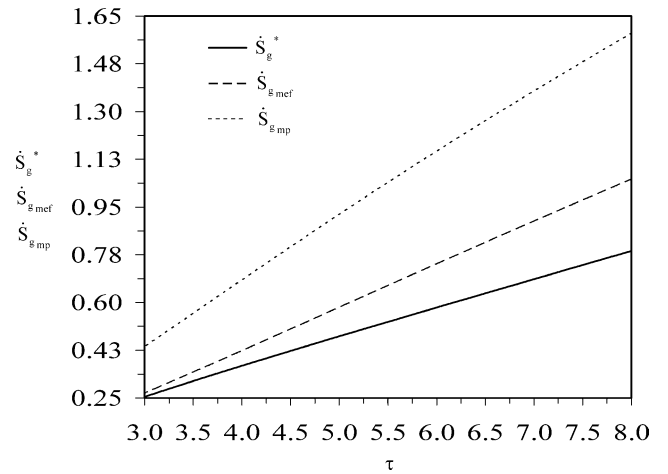


Fig. 8. The effect of source temperature ratio  $\tau$ , on the optimal entropy generation rates ( $T_L = 300$  K,  $T_H = 1200$  K,  $\xi = 0.02$ ,  $\eta_C = 0.9$ ,  $\eta_T = 0.95$ ,  $\varepsilon_L = \varepsilon_H = 0.9$ ).

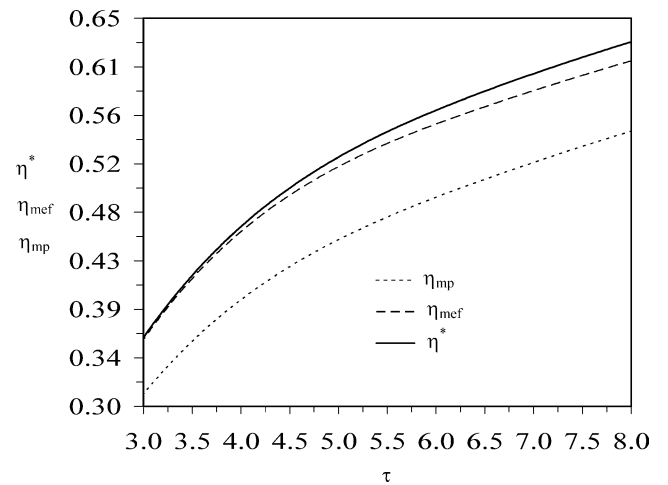


Fig. 9. The effect of source temperature ratio  $\tau$ , on the optimal thermal-efficiency values ( $T_L = 300$  K,  $T_H = 1200$  K,  $\xi = 0.02$ ,  $\eta_C = 0.9$ ,  $\eta_T = 0.95$ ,  $\varepsilon_L = \varepsilon_H = 0.9$ ).

ECOP function has the advantage of lower entropy generation rate together with higher thermal efficiency, although it has slightly lower power output in comparison to  $\dot{E}_{\max}$  and  $\dot{W}_{\max}$  conditions. The differences of the three compared performance curves presented in Figs. 7–9 become greater as  $\tau$  tends to increase.

#### 4. Conclusion

In this study, as an alternative to the ecological function found in the literature (since it may take negative values), a new objective function criterion which is dimensionless and always positive and also that can permit to evaluate the ecological performances of the heat engines, refrigerators and heat pumps on the same basis has been introduced and applied to gas turbines based on irreversible Brayton cycle model. The new thermo-ecological objective function, namely the ecological coefficient of performance (ECOP) is defined as the ratio of power to the loss rate of availability. In the analysis, the maximum of ECOP and the corresponding optimal performance and design parameters are derived analytically.

The obtained optimization results are interpreted by comparing them with those results of the maximum ecology and the maximum power criteria studied extensively in the literature in terms of entropy generation rate, thermal efficiency and the power output. The results of this study implies that a design parameters based on the maximum of ECOP objective function conditions may represent a compromise between the power output and the loss rate of availability for environmental aspects.

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